A MATHEMATICAL MODEL FOR CHYME FLOW IN THE SMALL INTESTINE DURING GASTROENTERITIS INFECTION

Supported by tetfund TERTIARY EDUCATION TRUST FUND

B. Disu[1]* and R. A. Kareem[2]
[1]Department of Mathematics, National Open University of Nigeria, Jabi, Abuja
[2]Department of Mathematics/Statistics, Lagos State Polytechnic, Ikorodu, Nigeria
*Corresponding author:adisu@noun.edu.ng

Received: April 07, 2017 **Accepted:** August 16, 2017

Abstract: A mathematical model is considered to study peristaltic transport of chyme in the small intestine during gastroenteritis infection. The Jeffrey model fluid flow is used for the peristaltic flow of chyme in the intestine. The assumptions of peristaltic rush waves and asymmetric channel of the inner part of the small intestine are made. The equations governing the flow are simplified by applied low Reynolds number and long wave approximation. Exact solutions are obtained for velocity and pressure rise. The numerical computations are presented graphically. It is found that the frictional parameter favour the forward movement of chyme through the small intestine during gastroenteritis infection.

Keywords: Chyme, gastroenteritis, Jeffrey, peristaltic flow, small intestine

Introduction

Chyme consists of water, hydrochloric acid, digestive enzymes and food-bolus, which passes from the stomach into the small intestine during metered (the periodic opening of the pyloric sphincter). It results from the mechanical and chemical breakdown of food-bolus in the stomach and moves slowly through the pyloric sphincter into the duodenum, where the transport of chyme along the small intestine begins. Depending on the quantity and contents of the meal, the stomach will ground the food-bolus into chyme between 40 min and a few hours (Akbar *et al.*, 2013). Once the process of mastication of food is completed, the stomach propels chyme through the pyloric sphincter into the small intestine.

The small intestine is the longest part of the gastrointestinal tract between the stomach and the large intestine. It has a single inlet and outlet, separated into three distinct regions: the duodenum, the jejunum, and the ileum. It can be described mathematically, as the longest, highly convoluted tube of about 6-7 m in length and has average radius of about 1.25 cm lying in the central laying in the central lower parts of abdomen (Guyton and Hall, 2006).

Digestion of chyme in the small intestine involves two processes (i) mechanical digestion, is the propulsion and mixing of chyme in the small intestine (ii) chemical digestion, responsible for the catabolic reactions that break down carbohydrates, proteins and lipids into small molecules that could be absorbed by the cell membranes. The movement of chyme along the small intestine is generated by (i) peristalsis, (ii) segmentation and (iii) pendular movements (Stavitsky*et al.*, 1981). Peristalsis and segmentation are basic electrical rhythm movements that cause contractions of the circular muscle layer of the intestinal wall. Peristaltic contractions spread along down the intestine whereas segmentation contractions are stationary and local (Monica, 2011). Pendular movements, on the other hand, cause retropulsion of lumen contents with a characteristic back and forth pattern.

Peristaltic waves propel chyme through the small intestine in about 3 - 6 h (Liu *et al.*, 2003) with an average velocity of 1.0 cm per min (Guyton and Hall, 2006). The waves are very weak and usually die out after travelling for about 3 - 5 cm, as a result the movement of chyme is very slow. Importantly, it spreads out (mixing) the content of the chyme along the intestine.

Normally, peristalsis in the small intestine is very weak, but a very strong irritation of the intestine mucosa occurs in cases of gastroenteritis. This causes powerful and rapid peristalsis called peristaltic rush. The powerful peristaltic contractions travel long distances in the small intestine within few minutes, sweeping the contents of the intestine into the colon and thereby relieving the small intestine of irritativechyme and excessive distention. This condition is called hyper-motility or over-activity of the small intestine.

Gastroenteritis is an intestinal infection which causes inflammation of the lining of the small intestine. It may be as a result of poor hygiene, contact with animals, consumed food or water that has been contaminated with bacteria and hormonal changes during menstruation. There are different types of bacteria that can cause gastroenteritis namely: *Staphylococcus*, *Salmonella*, *Yersinia*, *Shigella*, *Campylobacter*, *E. coli*, etc. The symptoms of the gastroenteritis include: diarrhea, abdominal pains and cramps, loss of appetite, nausea and vomiting, blood in stools and fever.

The greatest danger ensuing from this infection is dehydration. The potential danger of dehydration is greatest among children. This represents a large percentage of the problems confronting the public health care both in developed and developing countries. The World Health Organization (WHO) estimated that in developed countries, up to 30% of the population suffers from gastroenteritis yearly, while in developing countries up to two (2) million deaths are estimated per year (World Health Organization, 2007).

It thus becomes necessary to study the Mathematical structure of the movement of chyme along the small intestine during infection of Gastroenteritis. A Mathematical understanding of the structure of the infected small intestine may provide useful information about the dehydration problems among children and adults particularly in the developing countries.

In view of the above discussions, Mathematical studies of the peristaltic transport of chyme in the small intestine have been have carried out by many researchers (Akbar *et al.*, 2013, Riahi and Roy, 2011; Tripathi, 2012). Most of the researchers, studied peristaltic flow of chyme in the small intestine using Newtonian/non-Newton fluid model which does not possess physical properties of chyme. Physically, chyme can be described as semi-fluid which possesses both viscous and elastic properties. The Jeffery model is the viscoelastic model which constitutes viscous and elastic characteristics in the formulation of the semi – fluid. For examples, bread, fruit jam and almost edible semi-solids bear both viscous and elastics properties which can be simulated using the Jeffery viscoelastic model (Tripathi*et al.*, 2013).

Haya*et al.* (2006) studied peristaltic flow in a tube with an endoscope using Jeffrey viscoelastic model. Hayat and Ali (2008) investigated peristaltic motion a Jeffrey fluid under the effect of magnetic field in a tube. Kothandapani and Srinivas (2008) studied peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel. Also,

Nadeem and Akam (2010) studied slip effects on the peristaltic flow of a Jeffrey fluid in an asymmetric channel under the effect of induced magnetic field. The authors discussed the effect of magnetic field, relaxation and retardation time on the peristaltic transport.
Motivated by the aforementioned studies and in view to increase understanding of peristaltic transport of chyme in the small intestine, we present a mathematical model for the peristaltic flow of chyme in the gastroenteritis infected small intestine. Jeffrey fluid model is used for chyme and asymmetric channel is used for the channel of flow in the small intestine.

Mathematical formulation

Consider peristaltic flow of a Jeffrey fluid in a two dimensional asymmetric channel with flexible walls. Here, the small intestine is treated as an asymmetric channel and the chyme passing through it as a Jeffrey fluid. The walls of channel generate strong peristaltic waves which provide means of transporting Jeffrey fluid with speed c in the channel. The upper and lower walls of the channel are represented by $Y = H_1 = d_1 + a_1 \cos\left(\frac{2\pi}{\lambda}(X-ct)\right)$and $Y = H_1 = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X-ct)+\phi\right)$, respectively (Fig. 1).

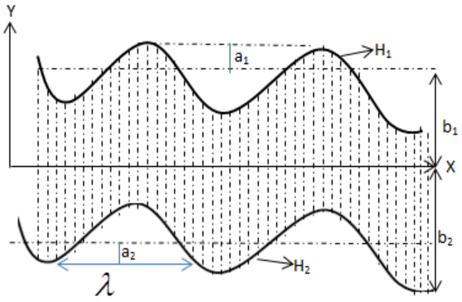

Fig. 1: Schematic diagram of a two dimensional asymmetric channel

wherea_1 and b_1 are the amplitudes, λ is the wave length, $d_1 + d_2$ is the width of the channel, ϕ is the phase difference $(0 \le \phi \le \pi)$ of the waves, $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ corresponds to the waves in phase, quantities a_1, b_1, d_1, d_2 and ϕ are such that,

$a_1^{\,2} + b_1^{\,2} + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2$. (1)

Therefore, the governing equations of Jeffrey fluid through the channel are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (2)$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U = -\frac{\partial P}{\partial X} + \frac{\partial \tau_{XX}}{\partial X} + \frac{\partial \tau_{XY}}{\partial Y} - \frac{\mu}{K}U, \qquad (3)$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V = -\frac{\partial P}{\partial Y} + \frac{\partial \tau_{XY}}{\partial X} + \frac{\partial \tau_{YY}}{\partial Y} - \frac{\mu}{K}V, \qquad (4)$$

whereρ is the density, μ is the dynamics viscosity, P is the pressure, U and V are the velocity components in $X-$ and $Y-$directions in a fixed frame.
The constitutive equation for the stress tensor S for a Jeffrey fluid is defined by (Srinivas and Kothandapani, 2008)

$$\tau = \frac{1}{1+\Lambda_1}(\dot{\gamma} + \Lambda_2 \ddot{\gamma}), \qquad (5)$$

whereΛ_1 and Λ_2 are frictional parameter.
Introducing a wave frame $(\bar{x}, \bar{y})$ with moving with velocity c away from the fixed frame (X, Y) by the transformation as follows:

$\bar{x} = X - ct,\quad \bar{y} = Y,\quad \bar{u} = U - c,\quad \bar{v} = V,\quad \bar{p} = P(X,t).$
(6) Substituting Eq. (6) into Eqs. (2) - (5), we obtain

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \qquad (7)$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \tau_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \tau_{\bar{x}\bar{y}}}{\partial \bar{y}} - \frac{\mu}{K}(\bar{u} + c), \qquad (8)$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{v} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \tau_{\bar{x}\bar{y}}}{\partial \bar{x}} + \frac{\partial \tau_{\bar{y}\bar{y}}}{\partial \bar{y}} - \frac{\mu}{K}\bar{v}, \qquad (9)$$

where,

$$\tau_{\bar{x}\bar{x}} = \frac{1}{1+\Lambda_1}\left[1 + \Lambda_2\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\frac{\partial \bar{u}}{\partial \bar{y}}\right], \qquad (10)$$

$$\tau_{\bar{x}\bar{y}} = \frac{1}{1+\Lambda_1}\left[1 + \Lambda_2\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\right]\left[\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}}\right], \qquad (11)$$

$$\tau_{\bar{y}\bar{y}} = \frac{1}{1+\Lambda_1}\left[1 + \Lambda_2\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\frac{\partial \bar{v}}{\partial \bar{y}}\right]. \qquad (12)$$

The followings are dimensionless variables

$$\bar{x} = \frac{\lambda x}{2\pi},\quad \bar{y} = ay,\quad \bar{u} = cu,\quad \bar{v} = cv,\quad \tau = \frac{\mu c}{a}T,\quad \bar{p} = \frac{\lambda \mu c}{2\pi a^2}p,$$
$$\bar{h} = ah \qquad (13)$$
$$k = \frac{K}{\mu a^2}, \delta = \frac{2\pi a}{\lambda},\quad Re = \frac{\rho c a}{\mu},\quad \lambda_1 = \frac{c\Lambda_1}{a},\quad \lambda_2 = \frac{c\Lambda_2}{a}$$

whereδ is the wave length and Re is Reynolds number.
Using dimensionless variables Eq. (13) into Eqs. (7) – (12), yield

$$\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (14)$$

$$Re\left[\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u\right] = -\frac{\partial p}{\partial x} + \delta\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} - \frac{1}{k}(u+1), \qquad (16)$$

$$Re\delta\left[\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)v\right] = -\frac{\partial p}{\partial y} + \delta\frac{\partial T_{xy}}{\partial x} + \delta^2\frac{\partial T_{yy}}{\partial y} - \frac{1}{k}v \qquad (17)$$

$$T_{xx} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\delta\frac{\partial u}{\partial x}\right] \qquad (18)$$

$$T_{xy} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\right]\left[\frac{\partial u}{\partial y} - \delta\frac{\partial v}{\partial x}\right], \qquad (19)$$

$$T_{yy} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\right]\left[\delta\frac{\partial v}{\partial y}\right] \qquad (20)$$

Defining a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\delta\frac{\partial \psi}{\partial x} \qquad (21)$$

Eq.(21) is identically satisfied and (14) – (20) become

$$Re\delta\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial \psi}{\partial y}\right] = -\frac{\partial p}{\partial x} + \delta\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} - \frac{1}{k}\left(\frac{\partial \psi}{\partial y} + 1\right) \qquad (22)$$

$$-Re\delta^3\left[\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial \psi}{\partial x}\right] = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial T_{xy}}{\partial x} + \delta\frac{\partial T_{yy}}{\partial y} + \frac{1}{k}\delta\frac{\partial \psi}{\partial x} \qquad (23)$$

where,

$$T_{xx} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\delta\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\delta\frac{\partial^2 \psi}{\partial x \partial y} \qquad (24)$$

$$T_{xy} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\delta\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\left[\frac{\partial^2 \psi}{\partial y^2} - \delta^2\frac{\partial^2 \psi}{\partial x^2}\right] \qquad (25)$$

$$T_{yy} = \frac{1}{1+\lambda_1}\left[1 + \lambda_2\delta\left(\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right)\right]\left[\delta^2\frac{\partial^2 \psi}{\partial x \partial y}\right] \qquad (26)$$

The corresponding boundary conditions are

$\psi = \frac{q}{2}$, at$y = h_1 = 1 + a\cos(2\pi x)$, $\psi = -\frac{q}{2}$, at $y = h_2 = -d - b\cos(2\pi x + \phi)$

$\frac{\partial \psi}{\partial y} = -1$, at$y = h_1$, $\frac{\partial \psi}{\partial y} = -1$, at $y = h_2$ (27)

where$q$is the flux in the wave frame and a, b, d and ϕ satisfied

$a^2 + b^2 + 2ab\cos\phi \le (1+d)^2$.

Under assumption Low Reynolds number and long wave length, Eqs.(22) – (26) become

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{1+\lambda_1}\frac{\partial T_{xy}}{\partial y} - \frac{1}{k}\left(\frac{\partial \psi}{\partial y} + 1\right) \qquad (28)$$

$$0 = -\frac{\partial p}{\partial y} \qquad (29)$$

$$T_{xx} = 0 \qquad (30)$$

$$T_{xy} = \frac{1}{1+\lambda_1}\left(\frac{\partial^2 \psi}{\partial y^2}\right) \qquad (31)$$

$$T_{yy} = 0 \qquad (32)$$

Elimination of the pressure p from Eqs. (28) and (29) by cross differentiation, we obtain the compatibility equation as follows;

$$\frac{\partial^4 \psi}{\partial y^4} - \frac{1+\lambda_1}{k}\left(\frac{\partial^2 \psi}{\partial y^2}\right) = 0 \qquad (33)$$

Solution of the problem

Solving the Eqs. (33) with boundary conditions (27), we obtain

$$\psi(y) = c_1 + c_2 + c_3 e^{my} + c_4 e^{-my} \qquad (34)$$

where$m = \sqrt{\frac{1+\lambda_1}{k}}$,

$$c_1 = -\frac{1}{2}\frac{(h_1+h_2)\{mq(e^{-m(h_1-h_2)} - e^{m(h_1-h_2)}) - 2(e^{-m(h_1-h_2)} - e^{m(h_1-h_2)}}{m(h_1-h_2)(e^{-m(h_1-h_2)} - e^{m(h_1-h_2)}) + 2(e^{-m(h_1-h_2)} - e^{m(h_1-h_2)}}$$

$$c_2 = -\frac{mq(e^{mh_2}e^{-mh_1}-e^{-mh_2}e^{mh_1})+2(e^{mh_2}e^{-mh_2}-e^{-mh_1}e^{mh_1})-2(e^{-mh_2}e^{mh_2}-e^{-mh_1}e^{mh_1})}{m(h_1-h_2)(e^{-mh_2}e^{mh_1}-e^{-mh_1}e^{mh_2})+2(e^{-mh_2}e^{mh_2}-e^{-mh_2}e^{mh_1})-2(e^{-mh_1}e^{mh_2}-e^{-mh_1}e^{mh_1})},$$

$$c_3 = -\frac{(e^{mh_2}-e^{mh_1})(q-(h_1-h_2))}{m(h_1+h_2)(e^{-mh_2}e^{mh_1}-e^{-mh_1}e^{mh_2})+2(e^{mh_2}-e^{-mh_1})(e^{-mh_2}-e^{-mh_1})},$$

$$c_4 = -\frac{(e^{mh_2}-e^{mh_1})(q-(h_1-h_2))}{m(h_1+h_2)(e^{-mh_2}e^{mh_1}-e^{-mh_1}e^{mh_2})+2(e^{mh_2}-e^{-mh_1})(e^{-mh_2}-e^{-mh_1})}.$$

The flux at axial any station in the fixed frame is

$$\bar{Q} = \int_{h_1}^{h_2}(u+1)\,dy = q + h_1 - h_2. \qquad (35)$$

The average flow rate over one period $\left(T = \frac{\lambda}{c}\right)$ of the peristaltic wave is given by

$$Q = \frac{1}{T}\int_0^T \bar{Q}\,dt = \frac{1}{T}\int_0^T (q + h_1 - h_2)dt = q + 1 + d. \quad (36)$$

The pressure gradient is obtained from Eq. (28) as

$$\frac{\partial p}{\partial x} = \frac{1}{1+\lambda_1}\frac{\partial^3\psi}{\partial y^3} - \frac{1}{k}\left(\frac{\partial\psi}{\partial y} + 1\right). \qquad (37)$$

Substituting Eq.(34) into Eq.(37), Eq.(37) becomes

$$\frac{\partial p}{\partial x} = m^3(c_4e^{my} - c_3e^{-my}) - m(c_4e^{my} - c_3e^{-my} - c_2 + 1) \quad (38)$$

It is of interest to calculate the pressure rise (Δp) over one wavelength as

$$\Delta p = \int_0^1\left(m^3(c_4e^{my} - c_3e^{-my}) - m(c_4e^{my} - c_3e^{-my} - c_2 + 1)\right)dx \quad (39)$$

Discussion of Results

Figs. 2 – 4 illustrate the variation of the volumetric flow rate of peristaltic wave with pressure gradient for different values of the phase difference, frictional parameter and permeability parameter.

Fig. 2 depicts the pressure gradient Δp against averaged volumetric flow rate Q for different values of phase difference ϕ. It is observed that in the range of values of the pressure gradient in the entire pumping region $(\Delta p > 0)$ and free pumping region $(\Delta p = 0)$, the volumetric flow rate decreases with the increase in values of the phase difference. However, in the co-pumping region$(\Delta p < 0)$, the volumetric flow rate increase with increase in magnitude of the phase difference. It means that the pressure rise requires small values of phase difference for large volumetric flow rate in the pumping and free pumping region. This trend reverses in the co-pumping region as pressure drop. It implies that the stronger peristaltic waves in the channel width enhance the pressure rise, but reduce the volumetric flow rate in the pumping and free pumping region. It is also noted that the volumetric flow rate is high in the co-pumping region, where proper mixing up and absorption nutrient from chyme take place. As a result of this, the nutrient of chyme in the intestine sweep into the large intestine.

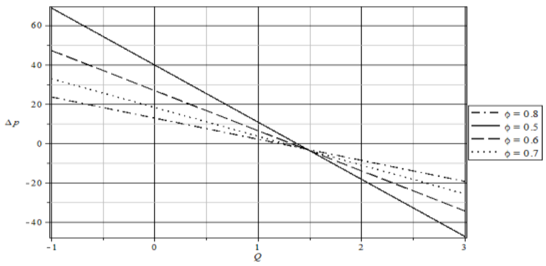

Fig. 2: Pressure vs averaged flow rate for different values of ϕ

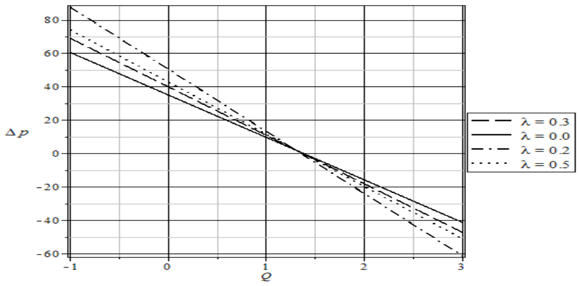

Fig. 3: Pressure vs averaged flow rate for different values of λ

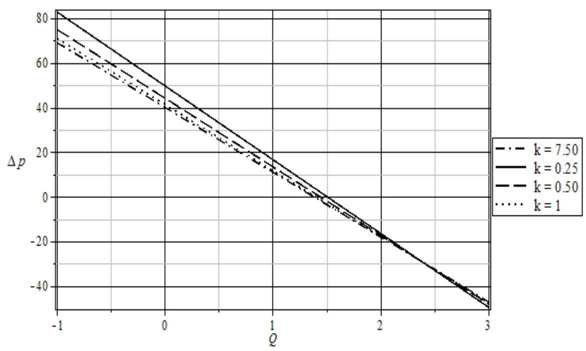

Fig. 4: Pressure vs averaged flow rate for different values of k

Figure 3 presents effect of frictional parameter $\lambda_1$on the pressure against average flow rate graph. It can be seen that an increase in values of λ_1 boosts the volumetric flow rate in the entire pumping region and free pumping region. Meanwhile, the volumetric flow rate reduces in the co-pumping region. This causes imbalance in the volumetric flow rate in the small intestine which result in abdominal pains. More so, the nutrient of chyme sweep into the large intestine without proper mixing up and absorption. Fig. 4 shows the influence of permeability parameter on the pressure gradient Δp against averaged volumetric flow rategraph. It can be seen that in the entire pumping, free pumping and co-pumping regions, the volumetric flow rate gradually reduce with increase in permeability parameter. This implies that absorption of nutrient from chyme take place; thereby reduce the rate of volumetric flow.

Conclusion

A mathematical model is developed for peristaltic transport of chyme through the small intestine. TheJeffrey fluid model is used for chyme and Darcy porous model is used for the absorption of nutrient in the small intestine. Under the assumption of low Reynolds number and long wave approximation, the equations governing the flow are simplified and solved analytically. The numerical solutions of the chyme flow are presented graphically. More so, the following observations are drawn from the results:

(i) The strong peristaltic waves cause improper mixing up and absorption nutrient from chyme.
(ii) The present of frictional parameter increases the volumetric flow rate of chyme in the small intestine;
(iii) The fractional parameter causes imbalance flow of chyme in the small intestine which may result in abdominal pain;
(iv) Increasing of permeability parameter gradually reduced the volumetric flow rate, which means the absorption of nutrient from chyme take place.

References

Akbar NS, Nadeem S & Lee C 2013. Characteristics of Jeffrey fluid model for peristaltic flow of chyme in the small intestine with magnetic field. *Journal of Results in Physics*, 3: 152-160.

Cheremisinoff NP 1988. *Encyclopedia of Fluid Mechanics, Rheology and Non-Newtonian Flows* (7), Gulf Publishing Co., Houston, TX.

Ebaid A 2008. Effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel. *Physics Letter A*, 372: 4493-4499.

Guyton AC & Hall JE 2006.*Textbook of Medical Physiology*. 12th ed. Philadelphia, PA: Saunders – Elsevier.

Hayat T, Ali N, Asghar S &SiddiquiAM 2006. Exact Peristaltic flow in tubes with an Endoscope.*Appl. Math. Comput.*,182: 359–368.

Hayat T &Ali N 2008.Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube.*Commun.Nonlinear Sci. Numer.Simul*., 13: 1343–1352.

Kothandapani M &Srinivas S 2008.Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel.*Int. J. Non-Linear Mech.,* 43: 915–924.

Liu L, Fishman ML, Kost J & Hicks KB 2003. Pectin-based systems for colon-specific drug delivery via oral route.*Biomaterials*, 24:3333-3343.

MisraJC&Pandey SK 1999.A Mathematical model for Oesophageal swallowing of a food-bolus.*Journal of Mathematical and Computer modelling, 33 (2001), 997 – 1009*.

Monica RJF 2011. *An engineering understanding of the small intestine*.PhD. Thesis, University of Birmingham.

Nadeem S &Akram S 2010. Slip effects on the peristaltic flow of a Jeffrey fluid in an asymmetric channel under the effect of induced magnetic field. *Int. J. NumericalMethods in Fluids*, 63(3): 374–394.

RiahiDN& Roy R 2011.Mathematics modelling of peristaltic flow of chyme in small intestine.*J. Applic. & Appl. Maths.*, 423 – 444.

Stavitsky D, MacagnoEO& Christensen J 1981.Finite-element analysis of flow induced by contractions like those of the intestine.*Journal of Biomechanics*, 14: 183.

Srinivas S &Kothandapani M 2008.Peristalic transport in an asymmetric channel with heat transfer.*A Note, Int. Commun. Heat Mass Transfer*, 35: 514-522.

Tripathi D, 2012. Peristaltic hemodynamic flow of couple-stress fluids through a porous medium with slip effect.*Trans. Porous Med.,* 92: 559.

Tripathi D, Pandey SK & Anwar Beg C 2013. Mathematical modelling of heat transfer effects on swallowing dynamics of viscoelastic food bolus through the human oesophagus. *Int. J. Thermal Sci.*, 70: 41-53.

World Health Organization 2007.Food safety and food borne illness and value chain management for food safety. “Forging Links between Agriculture and Health” CGIAR on Agriculture and Health Meeting in WHO/HQ Geneva.